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Algebraic characterizations of trace and decorated trace equivalences over tree-like structures ☆

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Abstract

Behavioural equivalences of labelled transition systems are characterized in terms of homomorphic transformations. This permits relying on algebraic techniques for proving systems properties and reduces equivalence checking of two systems to studying the relationships among the elements of their structures. Different algebraic characterizations of bisimulation-based equivalences in terms of particular transition system homomorphisms have been proposed in the literature. Here, it is shown that trace and decorated trace equivalences can neither be characterized in terms of transition system homomorphisms, nor be defined locally, i.e., only in terms of action sequences of bounded length and of root-preserving maps. However, results similar to those for bisimulation can be obtained for restricted classes of transition systems. For tree-like systems, we present the algebraic characterizations of *trace equivalence* and of three well-known decorated trace equivalences, namely *ready*, *ready trace equivalence* and *failure*. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

A possible approach to studying behavioural equivalences of labelled transition systems is that of characterizing them in terms of homomorphic transformations. In

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particular, we are interested in the transformations that are *locally defined*, i.e. that are defined only in terms of action sequences of bounded length. We say that a class X of homomorphisms *fully characterizes* a Y-equivalence over set $\mathscr{S}$ whenever any two systems in $\mathscr{S}$ are Y-equivalent if and only if they have a common image in $\mathscr{S}$ under X-homomorphism. This characterization permits relying on algebraic techniques for proving system properties and reduces equivalence checking of two systems to studying the relationships among the elements of their structures.

Given a behavioural equivalence, one may provide adequate conditions on transformations in order to preserve the equivalence or the modalities of the logics adequate for that equivalence. In [6], it has been shown that the class of *abstraction homomorphisms* (introduced in [7] to simplify labelled event structures) preserves, and actually fully characterizes, the strong and weak bisimulation equivalences of [19]. *Saturating homomorphisms* for a given logic are introduced in [1, 2], and are used to characterize a number of logically defined equivalences, such as strong bisimulation equivalence [19], the generalized transition system bisimulation characterized by Future Perfect logic [16] and branching bisimulation characterized by Hennessy–Milner logic extended with an "until" operator [12]. An account of the relationship between abstraction and saturating homomorphisms can be found in [3].

All the above-mentioned approaches have focused on bisimulation-based equivalences, or on their corresponding modal logics. Weaker than the bisimulation-based equivalences, decorated trace equivalences are a large family of equivalences that can be obtained via effective testing in the style of [11]. Among decorated trace equivalences, we will concentrate on those equivalences that rely on associating to each trace, say σ, the set of the actions that can be performed when at one of the states reachable via σ from the initial one. More specifically, we will first consider *trace equivalence* [17], then concentrate on *ready equivalence* [20], *ready trace equivalence* [4, 22], and *failure/testing equivalence* [5, 10]. Readers are referred to [13, 14] for an exhaustive overview and for a discussion of experimental settings that give rise to this kind of equivalences.

When looking for algebraic characterizations of decorated trace equivalences similar to those based on bisimulation, one should be aware of the fact that:

- trace and decorated trace equivalences are not inductively defined;
- the equivalence of two states depends both on their future capabilities and on their past sequences of actions (traces).

Due to these two features, we will see that trace and decorated trace equivalences over general labelled transition systems can be characterized neither in terms of transition system homomorphisms, nor in terms of locally defined root preserving maps. We will show however, that the wanted results can be obtained for restricted classes of transition systems, namely, mono-history transition systems and transition trees. *Mono-history transition systems* are directed acyclic labelled graphs where each node has a *unique access trace*; thus they may have nodes accessible via different paths but all paths connecting such nodes are labelled by the same trace. *Transition trees* are transition systems where each node has a *unique access path*. Over these restricted classes of transition systems, we define root preserving maps and homomorphisms to characterize trace equivalence, ready equivalence, ready trace equivalence and failure equivalence. We will thus exhibit:

(i) the class of surjective transition system homomorphisms which fully characterizes trace equivalence over mono-history transition systems;
(ii) the class of ready homomorphisms which fully characterizes ready and ready trace equivalences over mono-history transition systems and transition trees respectively;
(iii) the class of failure morphisms which fully characterizes failure equivalence over mono-history transition systems.

Indeed, we shall prove statements that are stronger than the above ones and will exhibit the following properties of the root preserving maps and homomorphisms:

– *Equivalence preservation*;
– *Possibility of standardization*;
– *Uniqueness of standardization*.

We shall restrict our attention to the strong variants of the equivalences, and consider only those systems whose actions are all visible. The generalization to systems with invisible actions, in the style of [19], is however straightforward.

The rest of the paper is organized as follow. Section 2 introduces the necessary notational background for labelled transition systems, and for trace and decorated trace equivalences. Section 3 shows that trace and decorated trace equivalences over general labelled transition systems can be characterized neither in terms of transition system homomorphisms nor in terms of locally defined root preserving maps. It then shows that, when restricted to mono-history transition systems, the class of surjective transition system homomorphisms fully characterizes trace equivalence. Section 4 introduces ready homomorphisms and proves that the class of ready homomorphisms fully characterizes ready equivalence over mono-history transition systems. Section 5 shows that when considering transition trees, the class of ready homomorphisms fully characterizes ready trace equivalence. Section 6 defines failure morphisms, a class of functions that are less demanding than homomorphisms, and shows that they can be used to characterize failure equivalence over mono-history transition systems. The last section contains a few concluding remarks.

2. Background and notations

In this section, we introduce the basic definitions for labelled transition systems, root preserving maps and homomorphism, together with those for the trace and decorated trace equivalences discussed in the paper, namely *trace equivalence* [17], *ready equivalence* [20], *ready trace equivalence* [4, 22], and *failure equivalence* [5, 10]. As already mentioned in the Introduction, we will restrict our attention to systems without silent moves.

Definition 2.1 (*Labelled transition systems*). A *labelled transition system* is a quadruple $\langle S, A, \rightarrow, s_0 \rangle$ where S is a countable set of states, A is a countable set of elementary actions, $\rightarrow \subseteq S \times A \times S$ is a set of transitions, and $s_0 \in S$ is the initial state.

A labelled transition system $\langle S, A, \rightarrow, s_0 \rangle$ is called *finitely branching*, if for any state s in S, set $\{(a, s') \mid (s, a, s') \in \rightarrow\}$ is finite. In this paper, we consider only labelled transition systems that are finitely branching.

In the following, we will use r, r', s, s', t, t' (possibly with index) to denote elements of S, and a, b, c (possibly with index) to denote elements of A. A^* will be used to denote the set of strings over A and σ (possibly with index) will denote one of its elements. $\mathscr{P}(A)$ will be used to denote the powerset of A. Moreover, a transition $(s, a, s') \in \rightarrow$ will be rendered as $s \xrightarrow{a} s'$ and labelled transition systems will be called transition systems for short. Furthermore, we will use the following conventions:
– $s \xrightarrow{a}$ will stand for $\exists s'$ such that $s \xrightarrow{a} s'$;
– $s \xrightarrow{\sigma} s_n$ where $\sigma = a_1 \ldots a_n$, will stand for $\exists s_1, \ldots, s_{n-1}$ such that $s \xrightarrow{a_1} s_1 \ldots \xrightarrow{a_n} s_n$;
– $s \xrightarrow{\sigma}$ will stand for $\exists s'$ such that $s \xrightarrow{\sigma} s'$;
– $s \not\xrightarrow{\sigma}$ will stand for *not* $s \xrightarrow{\sigma}$;
– a sequence of successive transitions will be called a *path*;
– $I(s)$ will be used to denote the set of initial actions from state $s : I(s) = \{a \in A \mid s \xrightarrow{a}\}$.
Root preserving maps for transition systems are those maps that preserve the initial states. Transition system homomorphisms are root preserving maps which also preserve the transition relations.

Definition 2.2 (*Root preserving maps*). Let $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s_0' \rangle$ be two transition systems.
– $h : S \rightarrow S'$ is a *root preserving map* if $h(s_0) = s_0'$.
– A root preserving map h is *surjective* if $h(S) = S'$.

Definition 2.3 (*Transition system homomorphisms* [2]). Let $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s_0' \rangle$ be two transition systems.
– $h : S \rightarrow S'$ is a *transition system homomorphism* if $h(s_0) = h(s_0')$ and $h(\rightarrow) \subseteq \rightarrow'$ where $h(\rightarrow) = \{h(s) \xrightarrow{a}' h(s') \mid s \xrightarrow{a} s'\}$;
– a homomorphism h is *surjective* if $h(S) = S'$.

Definition 2.4 (*Trace equivalence* [17]). (i) $\sigma \in A^*$ is a *trace* of state s, if there is a state s' such that $s \xrightarrow{\sigma} s'$.

(ii) If $Tr(s)$ denotes the set of traces of s, two transition systems $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s_0' \rangle$ are *trace equivalent* if $Tr(s_0) = Tr(s_0')$.

Definition 2.5 (*Ready equivalence* [20]). (i) A pair $\langle \sigma, X \rangle \in A^* \times \mathscr{P}(A)$ is a *ready pair* of state s, if there exists a state s' such that $s \xrightarrow{\sigma} s'$ and $I(s') = X$.

(ii) If $R(s)$ denotes the set of ready pairs of state s, two transition systems $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s_0' \rangle$ are *ready equivalent* if $R(s_0) = R(s_0')$.

The *failure semantics* is introduced in [5] and used in the construction of a model for the process algebra named CSP [18]. The impact of such an equivalence on labelled transition systems and its characterization as a testing equivalence are studied in [10].

Definition 2.6 (*Failure equivalence* [5]). (i) $\langle \sigma, X\rangle \in A^* \times \mathscr{P}(A)$ is a *failure pair* of state s, if there is a state s' such that $s \xrightarrow{\sigma} s'$ and $I(s') \cap X = \emptyset$.

(ii) If $F(s)$ denotes the set of failure pairs of s, two transition systems $T = \langle S, A, \rightarrow, s_0\rangle$ and $T' = \langle S', A, \rightarrow', s_0'\rangle$ are *failure equivalent* if $F(s_0) = F(s_0')$.

Definition 2.7 (*Ready trace equivalence* [4, 22]). (i) A sequence $X_0 a_1 X_1 a_2 \ldots a_n X_n \in \mathscr{P}(A) \times (A \times \mathscr{P}(A))^*$ is a *ready trace* of state s, if there exist states $s_1 \ldots s_n$ such that $s \xrightarrow{a_1} s_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} s_n$ with $I(s) = X_0$ and $I(s_i) = X_i$ for $i = 1, \ldots, n$.

(ii) If $RT(s)$ denotes the set of ready traces of state s, two transition systems $T = \langle S, A, \rightarrow, s_0\rangle$ and $T' = \langle S', A, \rightarrow', s_0'\rangle$ are *ready trace equivalent* if $RT(s_0) = RT(s_0')$.

Definition 2.8 (*Strong bisimulation* [21]). (i) A *bisimulation* between two transition systems $T = \langle S, A, \rightarrow, s_0\rangle$ and $T' = \langle S', A, \rightarrow', s_0'\rangle$ is a binary relation $U \subseteq S \times S'$ such that

(i) $s_0 U s_0'$;
(ii) $\forall s \in S,\ \exists s' \in S' : sUs'$ and $\forall s' \in S',\ \exists s \in S : sUs'$;
(iii) if sUt then, $\forall a \in A$:
  (a) $s \xrightarrow{a} s'$ implies $\exists t' : t \xrightarrow{a}{}' t'$ and $s'Ut'$;
  (b) $t \xrightarrow{a}{}' t'$ implies $\exists s' : s \xrightarrow{a} s'$ and $s'Ut'$.

(ii) We say T and T' are *bisimilar* if there is a bisimulation between them.

Definition 2.9 (*Abstraction homomorphisms* [6]). Let $T = \langle S, A, \rightarrow, s_0\rangle$ and $T' = \langle S', A, \rightarrow', s_0'\rangle$ be two transition systems. $h : S \rightarrow S'$ is called an *abstraction homomorphism* if it is a surjective transition system homomorphism satisfying:

$$\forall s_1 \in S, \quad a \in A,\ \ s_2' \in S'.\, h(s_1) \xrightarrow{a}{}' s_2' \textit{ implies } \exists s_2 \in S.\, s_1 \xrightarrow{a} s_2 \textit{ and } h(s_2) = s_2'.$$

The following theorem, due to Castellani [6], establishes a clear relation between bisimulation and abstraction homomorphism, formulated as in [2, 9].

Theorem 2.1 (Bisimulation and abstraction homomorphisms). *Two transition systems are bisimilar if and only if they have a common image under abstraction homomorphisms.*

As mentioned in the Introduction, we cannot obtain similar results for trace and decorated trace equivalences over general transition systems. This is possible only if we restrict our attention to subclasses of transition systems, namely to *mono-history transition systems* and *transition trees*. Mono-history transition systems are transition systems such that each state has a *unique access trace*, while transition trees are transition systems such that each state has a *unique access path*.

Definition 2.10 (*Mono-history transition systems and transition trees*). Let $T = \langle S, A, \rightarrow, s_0 \rangle$ be a labelled transition system.

(i) T is a *mono-history transition system* if each element of S is reachable from the initial state and whenever $s_0 \xrightarrow{\sigma_1} s$ and $s_0 \xrightarrow{\sigma_2} s$, we have $\sigma_1 = \sigma_2$.

(ii) T is a *transition tree* if each element of S is reachable from the initial state and whenever $s_0 \xrightarrow{a_1} r_1 \ldots \xrightarrow{a_n} r_n = s$ and $s_0 \xrightarrow{b_1} t_1 \ldots \xrightarrow{b_m} t_m = s$, we have $n = m$, $a_i = b_i$ and $r_i = t_i$, $i \in \{1, \ldots, n\}$.

When only mono-history transition systems or transition trees are considered, each state has a unique access trace. We will use $at(s)$ to denote the access trace of state s from the initial state s_0:

$$at(s) = \sigma \quad \text{iff } s_0 \xrightarrow{\sigma} s.$$

Definition 2.11 (*Equivalence preserving homomorphisms*). A class X of homomorphisms *preserves* a Y-equivalence (notation $\approx_Y$) over $\mathscr{S}$ if for any S_1, $S_2 \in \mathscr{S}$ and for any X-homomorphism $h : S_1 \rightarrow S_2$, we have $S_1 \approx_Y S_2$.

Definition 2.12 (*Algebraic characterizations of equivalences*). A class X of homomorphisms *fully characterizes* a Y-equivalence over $\mathscr{S}$ if any two systems in S are Y-equivalent if and only if they have a common image in S under X-homomorphism, i.e.,

(i) The class X of homomorphisms preserves Y-equivalence;

(ii) Any two Y-equivalent systems in S have a common image in S under X-homomorphism: $\forall S_1, S_2 \in \mathscr{S}$, $S_1 \approx_Y S_2$ *iff* $\exists S_3 \in \mathscr{S}$, $\exists X$-homomorphisms h_1, h_2, such that $h_i : S_i \rightarrow S_3$ $(i = 1, 2)$.

3. Trace equivalence of mono-history transition systems

As we know, the class of abstraction homomorphisms preserves strong bisimulation equivalence, and thus it preserves all the weaker equivalences as well. Obviously, for the latters, one would expect weaker conditions than those dictated by abstraction homomorphisms.

For example, the homomorphism of Fig. 1 preserves ready trace equivalence, failure equivalence, etc., but it does not satisfy the condition imposed by abstraction homomorphism: we have $h(s_4) \xrightarrow{a_2} h(s_2)$, but there does not exist t such that $s_4 \xrightarrow{a_2} t$ and $h(t) = h(s_2)$.

Indeed, an abstraction homomorphism is a transition system homomorphism with a strong additional restriction. When aiming at characterizing trace and decorated trace equivalences, we have to find other, less demanding, restrictions. However, if one con-

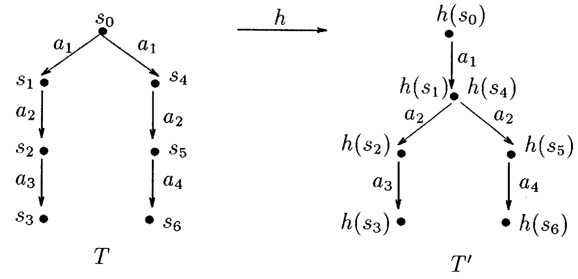

Fig. 1. A homomorphism preserving trace equivalence.

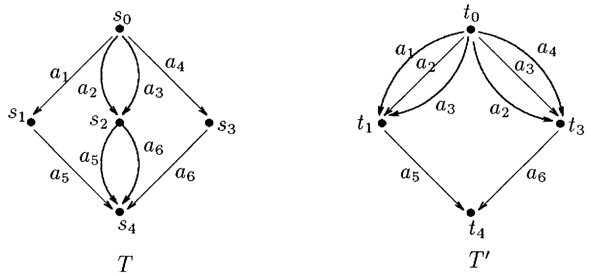

Fig. 2. A pair of trace equivalent processes with no common homomorphic image.

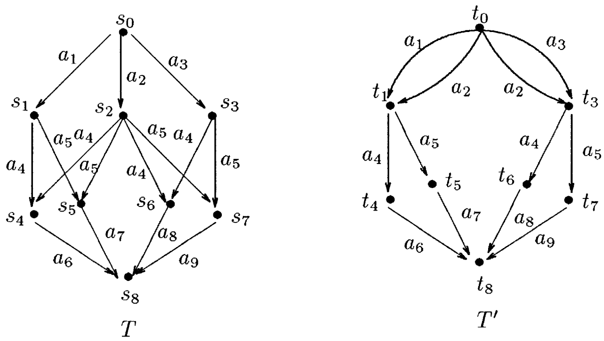

Fig. 3. A pair of ready equivalent processes with no common homomorphic image.

siders Fig. 2, he/she will realize that it is impossible to characterize trace equivalence over general (or even over acyclic) transition systems by simply adding restrictions on transition system homomorphisms: the required transformation may not preserve all transitions. T and T' in Fig. 2 are trace equivalent and T' contains the minimum number of states among all the transition systems that are trace equivalent to it. Thus, we may consider using T' as the common image of T and T', but there is no transformation from T to T' that preserves all transitions.

Also, Fig. 3 shows that it is impossible to characterize ready equivalence over acyclic transition systems by simply adding restrictions on homomorphisms. T and T' in Fig. 3

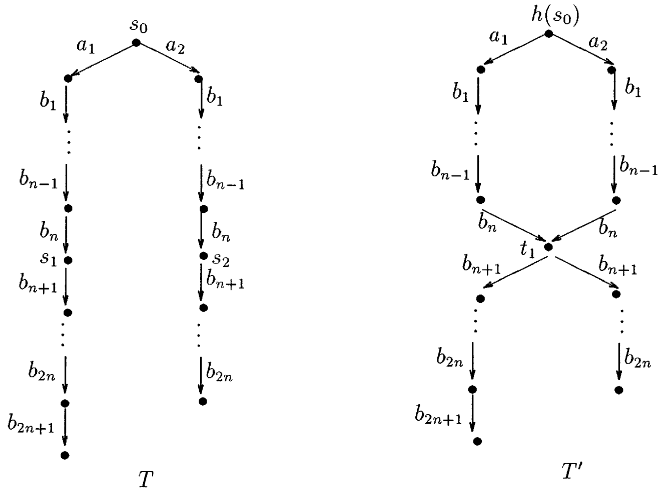

Fig. 4. Two processes that are not trace equivalent.

are ready equivalent and T' contains the minimum number of states among all transition systems ready equivalent to it. Thus, we may consider using T' as the common image for T and T', but there is no transformation from T to T' that preserves all transitions.

Thus, in order to achieve our goal, we would have to consider restrictions directly on general root preserving maps rather than on transition system homomorphisms. Furthermore, Fig. 4 shows that such restrictions may not be locally definable, i.e. defined only in terms of action sequences of bounded length. Let us consider T in Fig. 4 and the action sequences leading to s_1 and s_2 or starting from them. We could not differentiate between s_1 and s_2 if the action sequences (representing past and future behaviour) were bounded by n. But, the merging of s_1 and s_2 yields transition system T' that is not trace equivalent to T: the former contains $a_2b_1 \ldots b_{2n}b_{2n+1}$ while the latter does not.

The above considerations lead to the conclusion that the characterizations of trace and decorated trace equivalences over general or even acyclic transition systems might not exist or be so complex and distant from the elegance of the characterization of bisimulation to the point of becoming of limited interest.

We have, however, interesting results over tree-like structures: The key point of these results is that, over mono-history transition systems, the class of surjective transition system homomorphisms fully characterizes trace equivalence. Below we shall prove the following results for mono-history transition systems:

(i) *Equivalence preservation*: surjective transition system homomorphisms preserve trace equivalence over mono-history transition systems.
(ii) *Possibility of standardization*: Mono-history transition systems have a trace standard form relatively to surjective transition system homomorphisms.

(iii) *Uniqueness of standardization*: Two trace standard mono-history transition systems that are trace equivalent are isomorphic.

First of all, we show that surjective transition system homomorphisms preserve trace equivalence over mono-history transition systems:

Lemma 3.1 (Trace preservation). *Let* $T = \langle S, A, \rightarrow, s_0\rangle$ *and* $T' = \langle S', A, \rightarrow', s_0'\rangle$ *be two mono-history transition systems. If there exists a surjective transition system homomorphism* $h : S \rightarrow S'$, *then* $Tr(s_0) = Tr(h(s_0))$.

Proof. ($\subseteq$) $\sigma \in Tr(s_0)$ implies $\exists s$ s.t. $s_0 \xrightarrow{\sigma} s$. By definition of transition system homomorphism, it is easy to see that $h(s_0) \xrightarrow{\sigma}{}' h(s)$. So $\sigma \in Tr(h(s_0))$.

($\supseteq$) $\sigma \in Tr(h(s_0))$ implies $\exists s'$ s.t. $h(s_0) \xrightarrow{\sigma}{}' s'$. h is surjective implies $\exists s$ s.t. $h(s) = s'$. Since each node in T is reachable from s_0, $\exists \sigma'$ s.t. $s_0 \xrightarrow{\sigma'} s$. Then, by the definition of homomorphism, we have $h(s_0) \xrightarrow{\sigma'}{}' h(s)$. Since $h(s) = s'$ and T' is mono-historic, it follows that $\sigma' = \sigma$. Thus $\sigma \in Tr(s_0)$. $\square$

Next, we introduce the trace standard form and demonstrate that any mono-history transition system has a trace standard form under a surjective transition system homomorphism.

Definition 3.1. A mono-history transition system $\langle S, A, \rightarrow, s_0\rangle$ is called *trace standard* if it satisfies:

$$at(s_1) = at(s_2) \quad \text{implies } s_1 = s_2 \text{ for all } s_1, s_2 \in S.$$

Lemma 3.2 (Standardization). *Let* $T = \langle S, A, \rightarrow, s_0\rangle$ *be a mono-history transition system and define relation* $\sim_{\mathcal{T}} \subseteq S \times S$ *as* $\sim_{\mathcal{T}} (s_1, s_2)$ *iff* $at(s_1) = at(s_2)$, *then*:

(i) $\sim_{\mathcal{T}}$ *is an equivalence relation.*

(ii) *Let* S' *be the quotient of* S *w.r.t.* $\sim_{\mathcal{T}}$ *and let* $h : S \rightarrow S'$ *be the canonical surjective homomorphism. Then* $T' = \langle S', A, \rightarrow', h(s_0)\rangle$ *is trace standard.*

Proof. Proof for (1) is immediate. For (2) we prove that T' is a mono-history transition system. Then from the definition of $\sim_{\mathcal{T}}$, it follows immediately that T' is *trace standard*. In fact, we prove that T' is a tree: It is sufficient to demostrate that if $s_0 \xrightarrow{a_1} r_1 \ldots \xrightarrow{a_n} r_n$, $s_0 \xrightarrow{b_1} t_1 \ldots \xrightarrow{b_m} t_m$, and $\sim_{\mathcal{T}} (r_n, t_m)$, then we have $n = m$, $a_i = b_i$, $\sim_{\mathcal{T}} (r_i, t_i)$ for $i = 1, \ldots, n$.

Given $s_0 \xrightarrow{a_1} r_1 \cdots \xrightarrow{a_n} r_n$, $s_0 \xrightarrow{b_1} t_1 \ldots \xrightarrow{b_m} t_m$, and $\sim_{\mathcal{T}} (r_n, t_m)$, it is immediate by definition of $\sim_{\mathcal{T}}$ that $n = m$, $a_i = b_i$ for $i = 1, \ldots, n$. Now for any i where $1 \leqslant i \leqslant n$, we have $a_1 \ldots a_i = b_1 \ldots b_i$, which implies $\sim_{\mathcal{T}} (r_i, t_i)$. $\square$

Note that the h in the above lemma is a surjective transition system homomorphism by construction.

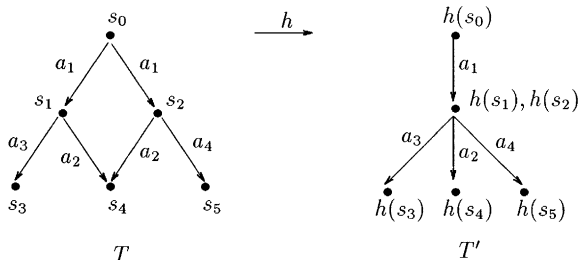

Fig. 5. A trace preserving transition system homomorphism.

Example 3.1. In Fig. 5, T' is the trace standard form of T, and h is a surjective transition system homomorphism.

Finally, we show uniqueness of trace standard form.

Lemma 3.3 (Uniqueness of standardization). *Any two trace standard mono-history transition systems that are trace equivalent are isomorphic.*

Proof. Let $T=\langle S,A,\longrightarrow,s_0\rangle$ and $T'=\langle S',A,\longrightarrow',s'_0\rangle$ be two mono-history transition systems, where T and T' are *trace standard* and trace equivalent. Define $h:S\longrightarrow S'$ as below:

(i) $h(s_0)=s'_0$;

(ii) $\forall s\neq s_0$, by the reachability of the states in T and the uniqueness of the access trace to s, $\exists!\ \sigma$ *s.t.* $s_0\xrightarrow{\sigma}s$. Since $Tr(s_0)=Tr(s'_0)$, $s'_0\xrightarrow{\sigma}{}'$. As T' is trace standard, $\exists!\ s'$ such that $s'_0\xrightarrow{\sigma}{}'s'$. Now define $h(s)=s'$.

Obviously, h is a total function. Now we demostrate that h is an isomorphism by proving that (i) $\forall s'\in S'$, $\exists s\in S.$ *s.t.* $h(s)=s'$; (ii) $h(s_1)=h(s_2)$ implies $s_1=s_2$; (iii) $s\xrightarrow{a}r$ iff $h(s)\xrightarrow{a}{}'h(r)$.

(i) $\forall s'\in S'$, $\exists s\in S.$ *s.t.* $h(s)=s'$. For s'_0, we have $h(s_0)=s'_0$. $\forall s'\neq s'_0$, by the reachability of the states in T', $\exists\ \sigma.\ s'_0\xrightarrow{\sigma}{}'s'$. Since $Tr(s_0)=Tr(s'_0)$, $\exists s.\ s_0\xrightarrow{\sigma}s$. By the definition of h, $h(s)=s'$.

(ii) $h(s_1)=h(s_2)$ implies $s_1=s_2$. By the construction of h, $at(s_1)=at(h(s_1))$, $at(s_2)=at(h(s_2))$. Now, $h(s_1)=h(s_2)$ implies $at(h(s_1))=at(h(s_2))$, so we have $at(s_1)=at(s_2)$, and thus $s_1=s_2$ since T is trace standard.

(iii) $s\xrightarrow{a}r$ iff $h(s)\xrightarrow{a}{}'h(r)$.
- Given $s_0\xrightarrow{a}r$, it is immediate by the definition of h that $s'_0\xrightarrow{a}{}'h(r)$.
- Given $s\xrightarrow{a}r$ where $s\neq s_0$, $\exists\sigma.$ *s.t.* $s_0\xrightarrow{\sigma}s\xrightarrow{a}r$. Since $Tr(s_0)=Tr(s'_0)$, $\exists s',r'$ s.t. $s'_0\xrightarrow{\sigma}{}'s'\xrightarrow{a}{}'r'$. By the definition of h, we have $h(s)=s'$, $h(r)=r'$, so $h(s)\xrightarrow{a}{}'h(r)$.
- Given $s'_0\xrightarrow{a}{}'h(r)$, by $Tr(s_0)=Tr(s'_0)$, $\exists r_1$ s.t. $s_0\xrightarrow{a}r_1$. By definition of h, $h(r_1)=h(r)$, so according to (ii), $r=r_1$. Thus $s_0\xrightarrow{a}r$.

– Given $h(s) \xrightarrow{a}' h(r)$ where $h(s) \neq s'_0$, $\exists \sigma$. *s.t.* $s'_0 \xrightarrow{\sigma}' h(s) \xrightarrow{a}' h(r)$. Since $Tr(s_0) = Tr(s'_0)$, $\exists s_1, r_1$ such that $s_0 \xrightarrow{\sigma} s_1 \xrightarrow{a} r_1$. By the definition of h, we have $h(s) = h(s_1)$, $h(r) = h(r_1)$. By (ii), $s = s_1$, $r = r_1$, so $s \xrightarrow{a} r$. $\square$

Theorem 3.1 (Characterization theorem for trace equivalence). *The class of surjective transition system homomorphisms fully characterizes trace equivalence over mono-history transition systems.*

Proof. Immediate from Lemmas 3.1–3.3. $\square$

The proof of Lemma 3.2 implies that the trace standard form of any mono-history transition system is a *transition tree*. Thus, we also have that the class of surjective transition system homomorphisms fully characterizes trace equivalence over transition trees.

4. Ready equivalence of mono-history transition systems

As we know from the previous section, the class of surjective transition system homomorphisms fully characterizes trace equivalence over mono-history transition systems. Ready equivalence is stronger than trace equivalence in the sense that we compare not only the traces in two systems, but also the final barbs of the traces. To extend the result for trace equivalence to ready equivalence, we should expect adding new conditions on transition system homomorphism. Such additional conditions should guarantee that $I(s) = I(h(s))$. According to homomorphism condition, we know that $s \xrightarrow{a}$ implies $h(s) \xrightarrow{a}'$; thus, to guarantee that $I(s) = I(h(s))$, we only need to require that $h(s) \xrightarrow{a}'$ implies $s \xrightarrow{a}$. We call a surjective transition system homomorphism with this additional condition *ready homomorphism*.

Definition 4.1 (*Ready homomorphisms*). Let $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s'_0 \rangle$ be two transition systems. A surjective transition system homomorphism $h : S \rightarrow S'$ is a *ready homomorphism* if it satisfies:

$$\forall s \in S, \quad a \in A.\ h(s) \xrightarrow{a}' \text{ implies } s \xrightarrow{a}.$$

Now we prove that the class of ready homomorphisms fully characterizes ready equivalence over mono-history transition systems. The proof follows the same pattern of the one in the previous section. First, we show that ready homomorphisms preserve ready equivalence over mono-history transition systems.

Lemma 4.1 (Ready equivalence preservation). *Let* $T = \langle S, A, \rightarrow, s_0 \rangle$ *and* $T' = \langle S', A, \rightarrow', s'_0 \rangle$ *be two mono-history transition systems. If there exists a ready homomorphism* $h : S \rightarrow S'$, *then* $R(s_0) = R(h(s_0))$.

Proof. Note that
(i) Since h is a homomorphism, $\forall s,\ a.\ s \xrightarrow{a}$ implies $h(s) \xrightarrow{a}{}'$, thus, we have $I(s) \subseteq I(h(s))$.
(ii) Since h is a *ready homomorphism*, $\forall s, a.\ h(s) \xrightarrow{a}{}'$ *implies* $s \xrightarrow{a}$, thus, we have $I(h(s)) \subseteq I(s)$.
Overall, $I(s) = I(h(s))$ for all $s \in S$. With this condition, the proof is analogous to that for Lemma 3.1. □

Next, we define ready standard form and demonstrate that any mono-history transition system has a ready standard form under a ready homomorphism. We proceed in two steps: *minimization* and *saturation*. First, we introduce *ready minimal form*.

Definition 4.2. A mono-history transition system is *ready minimal* if it satisfies

$$at(r) = at(s) \wedge I(r) = I(s) \quad \text{implies } r = s.$$

Lemma 4.2. *Let $T = \langle S, A, \rightarrow, s_0 \rangle$ be a mono-history transition system. Define relation $\sim_{\mathscr{R}} \subseteq S \times S$ as*

$$(s_1, s_2) \in\ \sim_{\mathscr{R}} \quad \textit{iff} \quad at(s_1) = at(s_2) \textit{ and } I(s_1) = I(s_2).$$

(i) $\sim_{\mathscr{R}}$ *is an equivalence relation*;
(ii) *If S' is the quotient of S w.r.t. $\sim_{\mathscr{R}}$, and $h : S \rightarrow S'$ is the canonical surjective homomorphism, then $T' = \langle S', A, \rightarrow', h(s_0) \rangle$ is ready minimal and h is a ready homomorphism.*

Proof. Proof of (1) is immediate. We prove 2.
- We show that T' is mono-historic, then by the definition of $\sim_{\mathscr{R}}$, it is easy to see that T' is *ready minimal*. According to the construction of h, $h(s_0) \xrightarrow{\sigma}{}' s'$ implies $\exists t_1 \in S$, *s.t.* $h(t_1) = s'$, $s_0 \xrightarrow{\sigma_1} t_1$. Similarly, $h(s_0) \xrightarrow{\sigma}{}' s'$ implies $\exists t_2 \in S$, s.t. $h(t_2) = s'$, $s_0 \xrightarrow{\sigma_2} t_2$. From $h(t_1) = h(t_2)$, i.e. $(t_1, t_2) \in\ \sim_{\mathscr{R}}$, we have $\sigma_1 = \sigma_2$.
- By the construction of h: $I(s) = I(h(s))$, so $h(s) \xrightarrow{a}{}'$ implies $s \xrightarrow{a}$. On the other hand, h is a surjective transition system homomorphism by construction. So h is a ready homomorphism. □

Fig. 6 shows two ready minimal mono-history transition systems. They are ready equivalent. The system in Fig. 7 can be seen as derived from s_0 or t_0 in Fig. 6, by adding two new transitions: $s_1 \xrightarrow{a_2} s_5$, $s_4 \xrightarrow{a_2} s_2$ (or correspondingly, $t_1 \xrightarrow{a_2} t_5$, $t_4 \xrightarrow{a_2} t_2$). This modification preserves ready equivalence. Indeed, this is the key idea of the construction of a homomorphism between s_0 and r_0, and between t_0 and r_0:

$$h_1(s_i) = r_i, \quad \text{for } i = 1, \ldots, 8,$$

$$h_2(t_2) = r_5, \quad h_2(t_5) = r_2, \quad h_2(t_i) = r_i, \quad \text{for } i = 0, 1, 3, 4, 6, 7, 8.$$

In fact, Fig. 7 can be considered as the common image of T and T' in Fig. 6.

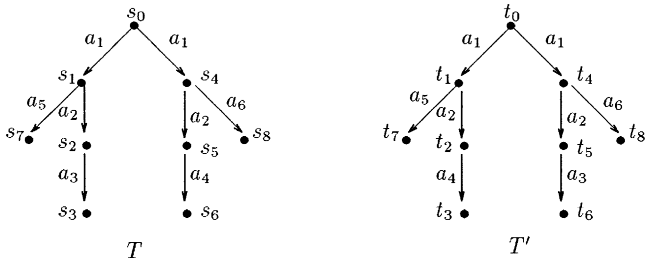

Fig. 6. Ready equivalent processes.

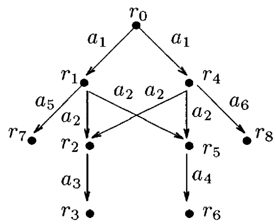

Fig. 7. Standard form for ready equivalence.

Given a mono-history transition system, its ready standard form is defined on its ready minimal form augmented by some necessary transitions in order to make it standard with respect to ready equivalence.

Definition 4.3. Let $T=\langle S,A,\rightarrow,s_0\rangle$ be a mono-history transition system. T is called *ready standard* if it is *ready minimal*, and

$$s \xrightarrow{a} \wedge\, at(s)=at(r) \wedge r \xrightarrow{a} t \quad \text{implies } s \xrightarrow{a} t.$$

Example 4.1. The system in Fig. 7 is *ready standard.*

Lemma 4.3 (Standardization). *Any mono-history transition system has a ready standard form under a ready homomorphism.*

Proof. Let $T=\langle S,A,\rightarrow,s_0\rangle$ be a mono-history transition system, S' be the quotient of S w.r.t. $\sim_{\mathscr{R}}$ as defined in Lemma 4.2, and $h:S\rightarrow S'$ be the canonical surjective homomorphism from T to $T1=\langle S',A,\rightarrow_1,s_0'\rangle$. Let $T2=\langle S',A,\rightarrow_2,s_0\rangle$ be defined by $g:S\rightarrow S'$ such that $g(s)=h(s)$ and $\rightarrow_2$ be the least transition relation satisfying
(i) $s \xrightarrow{a}_1 r$ implies $s \xrightarrow{a}_2 r$;
(ii) $s \xrightarrow{a}_1 \wedge\, at(s)=at(r) \wedge r \xrightarrow{a}_1 t$ implies $s \xrightarrow{a}_2 t$.
By Lemma 4.2, $T1$ is a minimal mono-history transition system. From the construction of $\rightarrow_2$, it can be easily seen that $T2$ remains a minimal mono-history transition system. And finally, by definition, $T2$ is ready standard. By Lemma 4.2, h is a ready homo-

morphism. Thus, according to the definition of g, it can be immediately concluded that g is a ready homomorphism. □

Now we show the uniqueness of ready standard form.

Lemma 4.4 (Uniqueness of standardization). *Any two ready standard mono-history transition systems that are ready equivalent are isomorphic.*

Proof. Let $T=\langle S,A,\rightarrow,s_0\rangle$ and $T'=\langle S',A,\rightarrow',s_0'\rangle$ be two mono-history transition systems, where T and T' are *ready standard* and ready equivalent. Define $h:S\rightarrow S'$ as below:

(i) $h(s_0)=s_0'$;

(ii) $\forall s\neq s_0$, by the reachability of the states in T and the uniqueness of the access trace to s, $\exists!\ \sigma$ s.t. $s_0\xrightarrow{\sigma}s$. Since $R(s_0)=R(s_0')$, and T' is ready standard, $\exists!\ s'$ such that $s_0'\xrightarrow{\sigma}{}'s'$ and $I(s')=I(s)$. Now define $h(s)=s'$.

Obviously, h is a total function. Now we show that $s\xrightarrow{a}r$ iff $h(s)\xrightarrow{a}{}'h(r)$. The other part of the proof is analogous to that in the proof of Lemma 3.3.

– Given $s_0\xrightarrow{a}r$, it is immediate by the definition of h that $s_0'\xrightarrow{a}h(r)$.
– Given $s\xrightarrow{a}r$ where $s\neq s_0$, $\exists\sigma$. such that $s_0\xrightarrow{\sigma}s\xrightarrow{a}r$. Since $R(s_0)=R(s_0')$, $\exists s',r',t'$, s.t. $s_0'\xrightarrow{\sigma}{}'s'$, $s_0'\xrightarrow{\sigma}{}'t'\xrightarrow{a}{}'r'$ where $I(s')=I(s)$, $I(r')=I(r)$. $I(s')=I(s)$ and $s\xrightarrow{a}r$ implies $s'\xrightarrow{a}{}'$. Since T' is ready standard, $t'\xrightarrow{a}{}'r'$ and $s'\xrightarrow{a}{}'$, we have $s'\xrightarrow{a}{}'r'$. By the definition of h, $h(s)=s'$, $h(r)=r'$. So $h(s)\xrightarrow{a}{}'h(r)$.
– Given $s_0'\xrightarrow{a}{}'h(r)$, since $R(s_0)=R(s_0')$, $\exists r_1$, *s.t.* $s_0\xrightarrow{a}r_1$, $I(r_1)=I(h(r))$. By the construction of h, $h(r_1)=h(r)$, and thus $r_1=r$. So $s_0\xrightarrow{a}r$.
– Given $h(s)\xrightarrow{a}{}'h(r)$ where $h(s)\neq s_0'$, $\exists\sigma$. *s.t.* $s_0'\xrightarrow{\sigma}{}'h(s)\xrightarrow{a}{}'h(r)$. Since $R(s_0)=R(s_0')$, we have that, $\exists s_1,r_1,t_1$, *s.t.* $s_0\xrightarrow{\sigma}s_1$, $s_0\xrightarrow{\sigma}t_1\xrightarrow{a}r_1$ where $I(s_1)=I(h(s))$, $I(r_1)=I(h(r))$. $I(s_1)=I(h(s))$ and $h(s)\xrightarrow{a}{}'h(r)$ implies $s_1\xrightarrow{a}$. Since T is ready standard, $t_1\xrightarrow{a}r_1$ and $s_1\xrightarrow{a}$ implies $s_1\xrightarrow{a}r_1$. By the definition of h, $h(s)=h(s_1)$, $h(r)=h(r_1)$. Thus $s=s_1$, $r=r_1$, and $s\xrightarrow{a}r$. □

Theorem 4.1 (Characterization theorem for ready equivalence). *The class of ready homomorphisms fully characterizes ready equivalence over mono-history transition systems.*

Proof. Immediate from Lemmas 4.1, 4.3, and 4.4. □

Note that, as shown in Figs. 6 and 7, the ready standard form of a transition tree is not necessarily a transition tree. As a consequence, we do not have the above result for transition trees.

5. Ready trace equivalence of transition trees

Now we proceed to examine ready trace equivalence. The class of ready homomorphisms does not fully characterize ready trace equivalence over mono-history transition systems. As an example, consider the homomorphism h from T in Fig. 6 to the system in Fig. 7 where $h(s_i) = r_i$ for $i = 1, \ldots, 8$; it is a ready homomorphism, but the two systems are not ready trace equivalent.

The main difference between ready and ready trace equivalence lies in the different placing of the requirements on the decorations: although both of them use $I(s)$ as decorations on s, in ready trace equivalence the requirement is put after each step of the trace while for ready equivalence, there are requirements only at the end of the trace. Thus, s_0 and t_0 in Fig. 6 are not ready trace equivalent, because $\{a_1\}a_1\{a_2, a_5\}a_2\{a_3\} \in RT(s_0)$, while we have $\{a_1\}a_1\{a_2, a_5\}a_2\{a_3\} \notin RT(t_0)$.

For ready equivalence, we only consider the final barb after sequence a_1a_2 and we have that $\langle a_1a_2, \{a_3\}\rangle$ is both in $R(s_0)$ and in $R(t_0)$.

More generally, putting decorations on each step of the trace is the key point that makes ready trace equivalence different, not only, from ready equivalence, but also, from trace and failure equivalence.

The ready homomorphism from T in Fig. 6 to the system in Fig. 7 indicates that in order to characterize ready trace equivalence, we may need to strengthen the condition

$$h(s) \xrightarrow{a}{}' \quad \text{implies } s \xrightarrow{a}.$$

by adding the relationship between the ending state of $h(s) \xrightarrow{a}{}'$ and that of $s \xrightarrow{a}$, but this leads us naturally to the abstraction homomorphism which is too strong. We leave open the problem of how to strengthen transition system homomorphism to characterize ready trace equivalence over mono-history transition system. Below, we restrict ourselves to a more restricted class of transition systems, i.e. transition trees, and show that the class of ready homomorphisms fully characterizes ready trace equivalence over this special kind of mono-history transition systems. The proof follows the same patterns of those in the previous sections.

First, we show that ready homomorphisms preserve ready trace equivalence over transition trees.

Lemma 5.1 (Ready trace preservation). *Let* $T = \langle S, A, \rightarrow, s_0\rangle$ *and* $T' = \langle S', A, \rightarrow', s_0'\rangle$ *be two transition trees. If there exists a ready homomorphism* $h : S \rightarrow S'$, *then* $RT(s_0) = RT(h(s_0))$.

Proof. Let $w = X_0a_1X_1a_2 \ldots a_nX_n$.

($\subseteq$) $w \in RT(s_0)$ implies $\exists s_1, \ldots, s_n$ such that $s_0 \xrightarrow{a_1} s_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} s_n$ and $I(s_i) = X_i$ for $i = 0, \ldots, n$. By definition of homomorphism, it is obvious that $h(s_0) \xrightarrow{a_1}{}' h(s_1) \xrightarrow{a_2}{}' \cdots \xrightarrow{a_n}{}' h(s_n)$; Like in the proof of Lemma 4.1, we have $I(s_i) = I(h(s_i))$ for $i = 0, \ldots, n$. Now $w \in RT(h(s_0))$ follows directly from the above.

($\supseteq$) $w \in RT(h(s_0))$ implies $\exists s_1', \ldots, s_n'$ with $h(s_0) \xrightarrow{a_1}{}' s_1' \xrightarrow{a_2}{}' \cdots \xrightarrow{a_n}{}' s_n'$, where $I(s_i') = X_i$, and $I(h(s_0)) = X_0$. Since h is surjective, $\exists s$ s.t. $h(s) = s_n'$. Since each node in T is reachable from s_0, $\exists b_1, \ldots, b_m, s_1, \ldots, s_{m-1}$, s.t. $s_0 \xrightarrow{b_1} s_1 \xrightarrow{b_2} \cdots \xrightarrow{b_m} s_m = s$. Then, by the definition of homomorphism, we have $h(s_0) \xrightarrow{b_1}{}' h(s_1) \xrightarrow{b_2}{}' \cdots \xrightarrow{b_m}{}' h(s_m) = h(s) = s_n'$. Since T' is a tree, $m = n$, $a_i = b_i$, $h(s_i) = s_i'$, for $i = 1, \ldots, n$. Like in the proof of

Lemma 4.1, we have $I(s_i)=I(h(s_i))$ so $I(s_i)=I(s_i')=X_i$, for $i=0,\dots,n$, and thus, $w\in RT(s_0)$. $\square$

Next, we introduce the ready trace standard form and demonstrate that any transition tree has a ready trace standard form under a ready homomorphism.

Definition 5.1. A transition tree $\langle S,A,\rightarrow,s_0\rangle$ is called *ready trace standard* if it satisfies

$$s\xrightarrow{a}r,\quad s\xrightarrow{a}t\quad\text{and}\quad I(r)=I(t)\quad\text{ implies } r=t.$$

Lemma 5.2 (Standardization). *Let $T=\langle S,A,\rightarrow,s_0\rangle$ be a transition tree, and let $\sim_{\mathscr{RT}}\subseteq S\times S$ be the least reflexive relation satisfying*:

$$(s_1,s_2)\in\sim_{\mathscr{RT}},\quad s_1\xrightarrow{a}t_1,\ s_2\xrightarrow{a}t_2,\quad I(t_1)=I(t_2)\quad\textit{ implies } (t_1,t_2)\in\sim_{\mathscr{RT}}.$$

(i) *$\sim_{\mathscr{RT}}$ is an equivalence relation.*
(ii) *Let S' be the quotient of S w.r.t. $\sim_{\mathscr{RT}}$. Let $h:S\rightarrow S'$ be the canonical surjective homomorphism. Then $T'=\langle S',A,\rightarrow',h(s_0)\rangle$ is ready trace standard and h is a ready homomorphism.*

Proof. (i) Obviously, $\sim_{\mathscr{RT}}$ is symmetric. We prove that it is also transitive. Given $(t_1,t_2)\in\sim_{\mathscr{RT}}$ and $(t_2,t_3)\in\sim_{\mathscr{RT}}$ where $t_1\neq t_2$, $t_2\neq t_3$, by the definition of $\sim_{\mathscr{RT}}$, $\exists s_1,s_2,s_2',s_3,a$ s.t.
– $(s_1,s_2)\in\sim_{\mathscr{RT}}$, $s_1\xrightarrow{a}t_1$, $s_2\xrightarrow{a}t_2$, $I(t_1)=I(t_2)$
– $(s_2',s_3)\in\sim_{\mathscr{RT}}$, $s_2'\xrightarrow{a}t_2$, $s_3\xrightarrow{a}t_3$, $I(t_2)=I(t_3)$.
This implies that $I(t_1)=I(t_3)$. On the other hand, since t_2 has unique access path, $s_2=s_2'$. So by induction: $(s_1,s_3)\in\sim_{\mathscr{RT}}$. But $(t_1,t_3)\in\sim_{\mathscr{RT}}$ by definition and thus $\sim_{\mathscr{RT}}$ is an equivalence relation.

(ii) We prove that T' is ready trace standard and h is a ready homomorphism.

(i) T' is ready trace standard. We prove that T' is a tree, then from the definition of $\sim_{\mathscr{RT}}$, it follows immediately that T' is *ready trace standard.* To demonstrate that T' is a tree, it is sufficient to show that if

$$s_0\xrightarrow{a_1}r_1\cdots\xrightarrow{a_n}r_n,\quad s_0\xrightarrow{b_1}t_1\cdots\xrightarrow{b_m}t_m,\quad\text{and}\quad\sim_{\mathscr{RT}}(r_n,t_m)$$

then we have

$$m=n,\quad a_i=b_i,\quad\sim_{\mathscr{RT}}(r_i,t_i)\quad\text{for } i=1,\dots,n.$$

The proof goes by induction on n. For $n=0$, by the definition of $\sim_{\mathscr{RT}}$, $m=0$ and the claim holds. For $n\geqslant 1$, by definition of $\sim_{\mathscr{RT}}$, $m\geqslant 1$. Now for $r_{n-1}\xrightarrow{a_n}r_n$, $t_{m-1}\xrightarrow{b_m}t_m$, according to the definition of $\sim_{\mathscr{RT}}$, $\sim_{\mathscr{RT}}(r_n,t_m)$ implies $a_n=b_m$ and $\sim_{\mathscr{RT}}(r_{n-1},t_{m-1})$. By induction hypothesis and $\sim_{\mathscr{RT}}(r_{n-1},t_{m-1})$, we have $n-1=m-1$, $a_i=b_i$, $\sim_{\mathscr{RT}}(r_i,t_i)$ for $i=1,\dots,n-1$. Thus, $n=m$, $a_i=b_i$, $\sim_{\mathscr{RT}}(r_i,t_i)$ for $i=1,\dots,n$.

(ii) h is a *ready homomorphism*. ($\forall s \in S.\ h(s) \xrightarrow{a}{}'$ implies $s \xrightarrow{a}$). From the construction of h, $h(s_1) \xrightarrow{a}{}'$ implies $\exists s_2.\ h(s_1) = h(s_2) \wedge s_2 \xrightarrow{a}$. By the definition of $\sim_{\mathscr{RT}}$, $\sim_{\mathscr{RT}}(s_1, s_2)$ implies $I(s_1) = I(s_2)$. Thus $h(s_1) = h(s_2)$ implies $I(s_1) = I(s_2)$ and so $s_2 \xrightarrow{a}$ implies $s_1 \xrightarrow{a}$. □

Finally, we show uniqueness of ready trace standard form.

Lemma 5.3 (Uniqueness of standardization). *Any two ready trace standard transition trees that are ready trace equivalent are isomorphic.*

Proof. Let $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s_0' \rangle$ be two transition trees, where T and T' are *ready trace standard* and ready trace equivalent. Define $h : S \rightarrow S'$ as below:
(i) $h(s_0) = s_0'$;
(ii) $\forall s \neq s_0$, by the reachability of the states in T and by the uniqueness of the access path to s, $\exists!\ s_1, \ldots, s_n, a_1, \ldots, a_n$ s.t. $s_0 \xrightarrow{a_1} \cdots \xrightarrow{a_n} s_n = s$. Since $RT(s_0) = RT(s_0')$, $I(s_0) a_1 I(s_1) \cdots a_n I(s_n) \in RT(s_0')$. Moreover, since T' is ready trace standard, $\exists!$ $s_1', \ldots, s_n'$ s.t. $s_0' \xrightarrow{a_1}{}' \cdots \xrightarrow{a_n}{}' s_n'$ where $I(s_i') = I(s_i)$ for $i = 1, \ldots, n$. Now define $h(s) = s_n'$.

Obviously, h is a total function. The proof to show that h is an isomorphism is analogous to the proof of Lemma 3.3. □

Theorem 5.1 (Characterization theorem for ready trace equivalence). *The class of ready homomorphisms fully characterizes ready trace equivalence over transition trees.*

Proof. Immediate from Lemmas 5.1–5.3. □

6. Failure equivalence of mono-history transition systems

Finally, we consider the algebraic characterization of failure equivalence. In this case, transition system homomorphisms are too demanding. We need to consider "unstructured" root preserving maps between the states. Fig. 8 is an evidence of this: As a mono-history transition system, T' has minimal set of states, and any non-trivial addition of transitions (without the addition of new states) will not preserve failure equivalence. Thus T' should be used as the common image of T and T'. However, a suitable root preserving map h from T to T' should have $h(s_5) = t_1$ or $h(s_5) = t_2$, but such an h cannot be a homomorphism: $h(s_5) \xrightarrow{a_2} h(s_3)$ and $h(s_5) \xrightarrow{a_3} h(s_4)$ cannot be present at the same time.

In the following, we discuss a class of failure morphisms which fully characterizes failure equivalence over mono-history transition systems.

The definition of failure morphism relies on predicate *mini*(_) defined below, that singles out those states s with the property that there exists no other state with the same access trace *at* as s but with a smaller set of next actions.

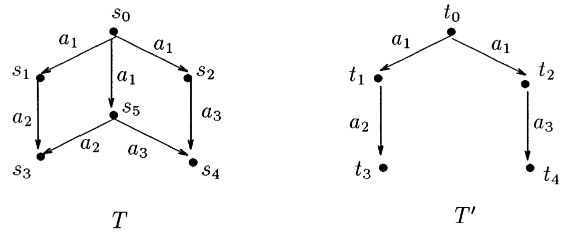

Fig. 8. Failure equivalent processes that are not homomorphic.

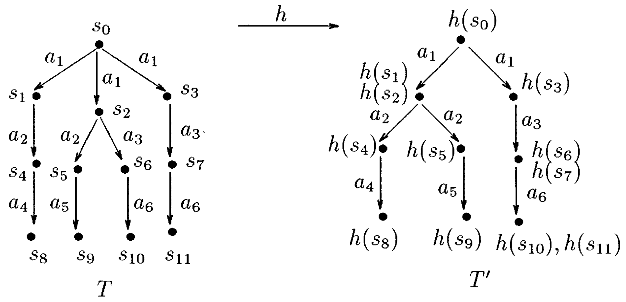

Fig. 9. A failure preserving minimization morphism.

Definition 6.1. $mini(s)$ iff $\nexists t$ such that $at(t) = at(s) \wedge I(t) \subset I(s)$.

Example 6.1. In Fig. 9, we have $mini(s_i)$ for $i = 0, 1, 3, \ldots, 11$, but we have also $\neg mini(s_2)$.

For failure equivalence, it is easy to see that predicate $mini(\_)$ singles out those states that are important to the minimization since they are essential for the failure test: the remaining states not satisfying $mini(\_)$ can be considered as redundant with respect to the failure test. *Failure morphism* introduced below preserves those states that satisfy $mini(\_)$, and the transitions related to them. It is worth remarking that condition 1 below follows from property

$$s \xrightarrow{a} t \quad \textit{implies } h(s) \xrightarrow{a} h(t) \qquad (*)$$

present in the definition of transition system homomorphisms. Here, since we consider general root preserving maps that may not enjoy property (*), we need condition 1.

Definition 6.2 (*Failure morphisms*). Let $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s'_0 \rangle$ be two mono-history transition systems. A surjective root preserving map $h : S \rightarrow S'$ is called *failure morphism* if it satisfies the following three conditions:

(i) $\forall s \in S.\ at(s) = at(h(s))$;

(ii) $\forall s \in S.\ mini(s)$ implies $(s \xrightarrow{a}$ *iff* $h(s) \xrightarrow{a}{}')$;

(iii) $\forall s' \in S'.\ mini(s')$ implies $\exists s \in S.\ h(s) = s' \wedge (s \xrightarrow{a}$ *iff* $h(s) \xrightarrow{a}{}')$.

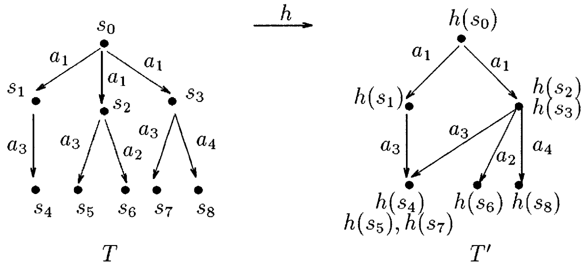

Fig. 10. A failure preserving minimization morphism.

Example 6.2. Figs. 9 and 10 provide two *failure morphisms*.

Now, we demonstrate that the class of failure morphisms fully characterizes failure equivalence over mono-history transition system. Again, the proof follows the same patterns of those in the previous sections. First, we show that failure morphisms preserve failure equivalence over mono-history transition systems:

Lemma 6.1 (Failure preservation). *Let $T=\langle S,A,\rightarrow,s_0\rangle$ and $T'=\langle S',A,\rightarrow',s'_0\rangle$ be two mono-history transition systems. If there exists a failure morphism $h:S\rightarrow S'$, then $F(s_0)=F(s'_0)$.*

Proof. We show that $F(s_0)=F(s'_0)$.

($\subseteq$) Given $\langle\sigma,X\rangle\in F(s_0)$ we have $\exists s\in S.\ s_0\xrightarrow{\sigma}s\wedge I(s)\cap X=\emptyset$.

If *mini*(s), by condition 2: $I(s)=I(h(s))$ and so $I(h(s))\cap X=\emptyset$. By condition 1 on the other hand, $at(h(s))=at(s)=\sigma$, i.e. $s'_0\xrightarrow{\sigma}{}'h(s)$, so $\langle\sigma,X\rangle\in F(s'_0)$.

If $\neg$*mini*(s), by definition of *mini*, $\exists t.\ at(t)=at(s)\wedge I(t)\subset I(s)$. Since the system is finitely branching, this implies $\exists t.\ at(t)=at(s)\wedge mini(t)\wedge I(t)\subset I(s)$. By *mini*$(t)$ and condition 2: $I(t)=I(h(t))$, thus $I(h(t))\subset I(s)$, and this implies that $I(h(t))\cap X=\emptyset$. By $at(t)=at(s)$ and condition 1, $at(h(t))=at(t)=at(s)=\sigma$, i.e. $s'_0\xrightarrow{\sigma}{}'h(t)$, and $\langle\sigma,X\rangle\in F(s'_0)$.

($\supseteq$) Given $\langle\sigma,X\rangle\in F(s'_0)$, we have $\exists s'\in S'.\ s'_0\xrightarrow{\sigma}{}'s'\wedge I(s')\cap X=\emptyset$.

If *mini*(s'), according to condition 3: $\exists s\in S.\ I(s)=I(s')\wedge h(s)=s'$. By $I(s)=I(s')$, we have $I(s)\cap X=\emptyset$. By $h(s)=s'$ and condition 1, $at(s)=at(h(s))=at(s')=\sigma$, i.e. $s_0\xrightarrow{\sigma}s$, and $\langle\sigma,X\rangle\in F(s_0)$.

If $\neg$*mini*(s'), by definition of predicate *mini*: $\exists t'\in S'.\ at(t')=at(s')\wedge I(t')\subset I(s')$, and since the system is finitely branching, this implies $\exists t'\in S'.\ at(t')=at(s')\wedge mini(t')\wedge I(t')\subset I(s')$. According to condition 3, $\exists t\in S.\ h(t)=t'\wedge I(t)=I(t')$. By $h(t)=t'$ and condition 1: $at(t)=at(h(t))=at(t')=at(s')=\sigma$, i.e. $s_0\xrightarrow{\sigma}t$. On the other hand, $I(t)=I(t')\subset I(s')$, so $I(t)\cap X=\emptyset$, and $\langle\sigma,X\rangle\in F(s_0)$. $\square$

Next, we define *failure standard form* and demonstrate that any mono-history transition system has a failure standard form under a failure morphism. As in the previous section, we proceed in two steps: *minimization* and *saturation*.

In order to introduce *failure minimal forms*, we need another auxiliary predicate *keytrace*(_). It captures those states s which, although not satisfying *mini*, have a next action which cannot be performed by any other state accessible via the same trace of s.

Definition 6.3. $keytrace(s)$ iff $\neg mini(s) \wedge \exists a.\ s \xrightarrow{a} \wedge \forall t \neq s.\ s.t.\ at(t) = at(s) : t \stackrel{a}{\not\longrightarrow}$.

The states singled out by *keytrace*(_) are not essential to the failure test (since they do not satisfy *mini*(_)), but they are essential for trace preservation. In Fig. 10, we have $keytrace(s_2)$ and $keytrace(s_3)$. In other words, although s_2 and s_3 can be considered redundant with respect to the failure test ($\neg mini(s_2)$, $\neg mini(s_3)$), they are essential to preserve traces a_1a_2 and a_1a_4 respectively. State s_2 in Fig. 9, on the other hand, is neither essential for the failure test ($\neg mini(s_2)$) nor essential for preserving the trace ($\neg keytrace(s_2)$).

An important feature of failure preserving minimization is that all states that are singled out by *keytrace*(_) with the same access trace, can be merged into a single state. For example, state s_2 and s_3 in Fig. 10 (with same access trace a) can be merged. The failure minimization is thus based on (i) maintaining states essential to failure test (*mini*) and (ii) merging states with the same access trace and are essential to preserving traces (*keytrace*).

Definition 6.4. A mono-history transition system is *failure minimal* if it satisfies the following three conditions:
(P1) $\forall s_1, s_2.\ at(s_1) = at(s_2) \wedge I(s_1) = I(s_2)$ implies $s_1 = s_2$;
(P2) $\forall s.\ mini(s) \vee keytrace(s)$;
(P3) $\forall s_1, s_2.\ at(s_1) = at(s_2) \wedge keytrace(s_1) \wedge keytrace(s_2)$ implies $s_1 = s_2$.

Example 6.3. The transition system in Figs. 9 and 10, the systems on the right sides are *failure minimal*, while the ones on the left are not.

Lemma 6.2. *Any mono-history transition system has a failure minimal form under a failure morphism.*

Proof. Given $T = \langle S, A, \rightarrow, s_0 \rangle$, let $S' = M_S \cup P_S$ where $M_S = \{m_{at(s), I(s)} \mid \exists s \in S, mini(s)\}$ and $P_S = \{p_{at(s)} \mid \exists s \in S,\ keytrace(s)\}$. Define $h : S \rightarrow S'$ as follows:
– if $mini(s)$, $h(s) = m_{at(s), I(s)}$;
– if $keytrace(s)$, $h(s) = p_{at(s)}$;
– if $\neg mini(s) \wedge \neg keytrace(s)$, $h(s) = h(t)$ for an arbitrary t such that $mini(t) \wedge at(s) = at(t) \wedge I(t) \subset I(s)$.
The transition relation is defined as the least relation satisfying:
– $(mini(s) \vee keytrace(s)) \wedge s \xrightarrow{a} s'$ *implies* $h(s) \xrightarrow{a}{}' h(s')$;
– $(\neg mini(s) \wedge \neg keytrace(s)) \wedge s \xrightarrow{a} s' \wedge mini(t) \wedge at(s) = at(t) \wedge t \xrightarrow{a}$ *implies* $h(t) \xrightarrow{a}{}' h(s')$.

Let $T' = \langle S', A, \to', h(s_0)\rangle$ be the transition system derived by h. Obviously, we have that T' is *failure minimal*. On the other hand, notice that *keytrace*(s) *implies* $\neg$*mini*(s), so h is a function, and it is easy to see that h is total and surjective. Now we show that h is a *failure morphism*.

(i) system T' is *mono-historic*, and $at(s) = at(h(s))$. It is sufficient to show that $s_0 \xrightarrow{\sigma_1} s_n \wedge h(s_0) \xrightarrow{a_2}{}' h(s_n)$ implies $\sigma_1 = \sigma_2$.

Let $s_0 \xrightarrow{l_1} s_{n-1} \xrightarrow{a_1} s_n,\ h(s_0) \xrightarrow{l_2}{}' s'_{n-1} \xrightarrow{a_2}{}' h(s_n)$ and consider the proof of $s'_{n-1} \xrightarrow{a_2}{}' h(s_n)$:

(a) $\exists r_{n-1},\ r_n$ such that $r_{n-1} \xrightarrow{a_2} r_n \wedge h(r_{n-1}) = s'_{n-1} \wedge h(r_n) = h(s_n)$. According to the construction of $h: h(r_n) = h(s_n)$ implies that we have $at(r_n) = at(s_n)$, thus $a_1 = a_2 \wedge at(r_{n-1}) = at(s_{n-1})$. Since $h(s_0) \xrightarrow{l_2}{}'\ h(r_{n-1}),\ l_1 = at(s_{n-1}) = at(r_{n-1})$, by induction we have $l_1 = l_2$ and so $\sigma_1 = \sigma_2$.

(b) $\exists r_{n-1},\ r_n,\ t: r_{n-1} \xrightarrow{a_2} r_n \wedge at(r_{n-1}) = at(t) \wedge h(t) = s'_{n-1} \wedge h(r_n) = h(s_n)$. According to the construction of $h: h(r_n) = h(s_n)$ implies that $at(r_n) = at(s_n)$, thus $a_1 = a_2 \wedge at(r_{n-1}) = at(s_{n-1})$. Since $at(t) = at(r_{n-1}): at(t) = at(s_{n-1}) = l_1$, by $h(s_0) \xrightarrow{l_2}{}' s'_{n-1} = h(t)$ and induction: $l_1 = l_2$. Thus, $\sigma_1 = \sigma_2$.

(ii) *mini*(s) implies $I(s) = I(h(s))$.

($\subseteq$) by the construction of the transition relation and of *mini*(s) we have that $\forall s, a.$ such that $s \xrightarrow{a}$ it holds that $h(s) \xrightarrow{a}{}'$.

($\supseteq$) by the construction: $h(s) \xrightarrow{a}{}'$ implies $\exists t.$ (*mini*(t) $\vee$ *keytrace*(t)) $\wedge$ $h(t) = h(s) \wedge t \xrightarrow{a}$. *mini*$(s)$ and $h(s) = h(t)$ implies $\neg$*keytrace*(t), thus we have *mini*(t). This, together with $h(s) = h(t)$, implies $t \xrightarrow{a}$.

(iii) $\forall s' \in S'.$ *mini*(s') implies $\exists s.\ h(s) = s' \wedge I(s) = I(h(s))$.

First of all, we show that *mini*(s') implies $s' \in M_S$. Suppose $s' \in P_S$. Then $\forall s \in S.\ h(s) = s'$, we have *keytrace*$(s)$, and by definition of h: $s \xrightarrow{a}$ *implies* $h(s) \xrightarrow{a}{}'$ i.e. $I(s) \subseteq I(h(s))$. On the other hand, *keytrace*(s) implies $\exists t.$ *mini*(t) $\wedge$ $at(t) = at(s) \wedge I(t) \subset I(s)$. By 2: $I(t) = I(h(t))$ so $I(h(t)) \subset I(s) \subseteq I(h(s))$, and by 1: $at(h(t)) = at(t),\ \ at(h(s)) = at(s)$, so $at(h(t)) = at(h(s))$. Thus $\neg$*mini*$(h(s))$ i.e. $\neg$*mini*(s') (contradiction). Now for $s' \in M_S$, by the construction, $\exists s.\ h(s) = s' \wedge$ *mini*(s), and according to (2): $I(s) = I(h(s))$. Hence, h is a *failure morphism*. $\square$

Once a mono-history transition system is failure minimized, the failure standard one can be obtained from it in the same way as that for ready standardization in the previous section.

Definition 6.5. A *mono-history* transition system is *failure standard* if it is *failure minimal* and satisfies the following two conditions:

(P4) *keytrace*$(s) \wedge at(r) = at(s) \wedge r \xrightarrow{a} r'$ implies $s \xrightarrow{a} r'$;

(P5) $s \xrightarrow{a} \wedge at(r) = at(s) \wedge r \xrightarrow{a} t$ implies $s \xrightarrow{a} t$.

Example 6.4. Fig. 7 is a *failure standard* one while T and T' in Fig. 6 are not.

Lemma 6.3 (Standardization). *Any mono-history transition system has a failure standard form under a failure morphism.*

Proof. Let $T=\langle S,A,\rightarrow,s_0\rangle$ be a *mono-history* transition system. According to Lemma 6.2, we have a *failure minimal* $T1=\langle S',A,\rightarrow_1,s'_0\rangle$ and a *failure morphism* $h:S\rightarrow S'$. Now let $T2=\langle S',A,\rightarrow_2,s_0{}'\rangle$ and $g:S\rightarrow S'$ such that $g(s)=h(s)$ and $\rightarrow_2$ is the least relation satisfying

– $r'\xrightarrow{a}_1 s'$ implies $r'\xrightarrow{a}_2 s'$;

– $\forall s'.\ keytrace(s')$ in T' implies

$$\forall r'\neq s' \text{ such that } at(r')=at(s')\wedge r'\xrightarrow{a}_1 t' \text{ we have } s'\xrightarrow{a}_2 t';$$

– $\forall s'.\ mini(s')$ in T' implies $\forall r'\neq s'$ such that $at(r')=at(s')\wedge r'\xrightarrow{a}_1 t'\wedge s'\xrightarrow{a}_1$ we have $s'\xrightarrow{a}_2 t'$.

By construction, we know that $T2$ is *failure standard*. Below, we demonstrate that g is a *failure morphism*.

(i) Since $T1$ is *mono-historic* and $at(h(s))=at(s)$, it is easy to see that $T2$ is still *mono-historic* and $at(g(s))=at(s)$.

(ii) Given $s\in S.\ s.t.\ mini(s)$, we have $s\xrightarrow{a}$ implies $h(s)\xrightarrow{a}_1$ by definition of h. Besides, according to the definition of the transition relation $\rightarrow_2$, we have $\xrightarrow{a}_1$ *implies* $s'\xrightarrow{a}_2$. Thus $s\xrightarrow{a}$ *implies* $g(s)\xrightarrow{a}_2$. On the other hand, by definition of h: $mini(s)$ implies $mini(h(s))$, so $h(s)\xrightarrow{a}_1$ implies $s\xrightarrow{a}$, while according to the definition of $\rightarrow_2$ we have $\forall s'\in S'.\ s'\xrightarrow{a}_2$ and $mini(s')$ in T' implies $s'\xrightarrow{a}_1$, thus $g(s)\xrightarrow{a}_2$ implies $s\xrightarrow{a}$.

(iii) $\forall s'\in S'.\ s.t.\ mini(s')$, since h is a *failure morphism*, $\exists s\in S.$ such that $I(s)=I(s')\wedge h(s)=s'$. Obviously, $mini(s)$, so according to (2): $I(g(s))=I(s)$, while we also have $g(s)=h(s)=s'$. $\square$

Finally, we show in Lemma 6.5 the uniqueness of failure standard form: any two failure standard mono-history transition systems that are failure equivalent are isomorphic. The proof of Lemma 6.5 is based on the following result.

Lemma 6.4. *Let $T=\langle S,A,\rightarrow,s_0\rangle$ and $T'=\langle S',A,\rightarrow',s'_0\rangle$ be two failure standard transition systems. If $F(s_0)=F(s'_0)$, then*

(i) *$\forall s\in S$ such that $mini(s)$. $\exists!\ s'\in S'$ such that $at(s)=at(s')\wedge I(s)=I(s')$, and $mini(s')$.*

(ii) *$\forall s\in S$ such that $keytrace(s)$. $\exists!\ s'\in S'$ such that $at(s)=at(s')\wedge I(s)=I(s')$, $keytrace(s')$.*

Proof. (i) $\forall s\in S.\ \langle at(s),\overline{I(s)}\rangle\in F(s_0)$. By $F(s_0)=F(s'_0):\langle at(s),\overline{I(s)}\rangle\in F(s'_0)$. This means $\exists s'\in S'.\ s'_0\xrightarrow{at(s)}{}'s'\wedge I(s')\cap\overline{I(s)}=\emptyset$, which implies $I(s')\subseteq I(s)$. Similarly, for $s'\in S':\exists s''\in S$ such that $s_0\xrightarrow{at(s)}s''\wedge I(s'')\subseteq I(s')$. Since $mini(s)$, for $I(s'')\subseteq I(s)$, we have $I(s'')=I(s')=I(s)$. T' is *failure standard*, so we know by (P1) that s' is unique. Now we show that $mini(s')$. Suppose $\neg mini(s')$. Then $\exists t\in S'$ such that $s'_0\xrightarrow{at(s)}{}'t\wedge I(t)\subset I(s')$.

Since $F(s_0) = F(s'_0)$, $s'_0 \xrightarrow{at(s)}{}' t$ implies $\exists r \in S$ such that $s_0 \xrightarrow{at(s)} r \wedge I(r) \subseteq I(t)$. Thus $I(r) \subset I(s') = I(s)$, which implies $\neg mini(s)$ (contradiction).

(ii) $keytrace(s)$ implies

$$\exists a. \text{ such that } s \xrightarrow{a} \quad \text{and} \quad \forall t \neq s \text{ and } mini(t) \wedge at(t) = at(s) : t \overset{a}{\not\longrightarrow} \qquad (*)$$

By $F(s_0) = F(s'_0)$, we have $T(s_0) = T(s'_0)$, where $T(s)$ denotes the set of traces of s. Thus $s_0 \xrightarrow{at(s)a}$ implies $s'_0 \xrightarrow{at(s)a}{}'$, i.e., $\exists s' \in S' : s'_0 \xrightarrow{at(s)}{}' s' \xrightarrow{a}{}'$. To show $keytrace(s')$, suppose $mini(s')$, by 1., $\exists t \in S : I(t) = I(s') \wedge at(t) = at(s') \wedge mini(t)$. But this contradicts $(*)$ because

(a) $at(t) = at(s') = at(s)$;

(b) $mini(t) \wedge keytrace(s)$ implies $t \neq s$;

(c) $I(t) = I(s') \wedge s' \xrightarrow{a}{}'$ implies $t \xrightarrow{a}$.

To show $I(s) = I(s')$, we just demonstrate that $I(s) \subseteq I(s')$. The vice versa part is symmetric. Given $s \xrightarrow{a}$, we have $at(s)a \in T(s_0)$, and thus $at(s)a \in T(s'_0)$, i.e. $\exists r' \in S'$. $s'_0 \xrightarrow{at(s)}{}' r' \xrightarrow{a}{}'$. Now $T2$ is *failure standard*, thus according to (P4), we have

$$at(r') = at(s) = at(s'), \quad r' \xrightarrow{a}{}', \; keytrace(s') \text{ implies } s' \xrightarrow{a}.$$

Hence, $\exists s' \in S' : keytrace(s') \wedge at(s') = at(s) \wedge I(s') = I(s)$, now since T' is *failure standard*, we can conclude by (P1) that s' is unique. □

Lemma 6.5 (Uniqueness of standardization). *Any two failure standard mono-history transition systems that are failure equivalent are isomorphic.*

Proof. Let $T = \langle S, A, \rightarrow, s_0 \rangle$ and $T' = \langle S', A, \rightarrow', s'_0 \rangle$ be two failure standard transition systems, where $F(s_0) = F(s'_0)$. Define $h : S \rightarrow S'$, where

$$h(s) = s' \quad \text{iff } at(s) = at(s') \wedge I(s) = I(s').$$

T and T' are *failure standard*, thus $\forall s \in S,\ s' \in S'$, we have

$$mini(s) \vee keytrace(s) \quad \text{and} \quad mini(s') \vee keytrace(s').$$

By Lemma 6.4, we conclude that h is one-to-one. To see that $s \xrightarrow{a} t \Leftrightarrow h(s) \xrightarrow{a}{}' h(t)$, we only demonstrate the "⇒" part: since h is one-to-one, the vice versa can be established symmetrically. Given $s \xrightarrow{a} t$, according to the definition of $h, at(t) = at(h(t))$, thus $\exists s' \in S'$. $s' \xrightarrow{a}{}' h(t) \wedge at(s) = at(s')$. On the other hand, since $at(s) = at(h(s))$, we have $at(s') = at(h(s))$.

keytrace$(h(s))$ Since $at(s') = at(h(s))$, $s' \xrightarrow{a}{}' h(t)$, by property (P4), we have $h(s) \xrightarrow{a}{}' h(t)$.

mini$(h(s))$ According to the definition of $h : I(s) = I(h(s))$, thus $h(s) \xrightarrow{a}{}'$. Now $at(s') = at(h(s))$, $s' \xrightarrow{a}{}' h(t)$, $h(s) \xrightarrow{a}{}'$, by property (P5), we have $h(s) \xrightarrow{a}{}' h(t)$. □

Theorem 6.1 (Characterization theorem for failure equivalence). *The class of failure morphisms fully characterizes failure equivalence over mono-history transition systems.*

Proof. Immediate by Lemmas 6.1, 6.3 and 6.5. □

Note again that, as shown in Figs. 6 and 7, the failure standard form of a transition tree may not be a transition tree. As a consequence, we do not have the above result over transition trees.

7. Conclusion and open problem

We have investigated the algebraic characterizations in terms of root preserving maps and homomorphisms of four behavioural equivalences over subclasses of Labelled Transition Systems (LTS). The considered equivalences are all weaker than the bisimulation based ones, for which similar results existed.

We have also argued that, when considering trace and decorated trace equivalences, the results obtained for bisimulation-based equivalences cannot be extended to general LTS by adding additional restrictions on transition system homomorphisms, and the restrictions that can be added to root preserving maps cannot be locally defined. Even though, the present paper constitutes a step forward for important subclasses of LTS, namely, tree-like structures, that are widely used to model nondeterministic computations.

Among the problems left open by our contribution, we would like to single out that of extending our results to failure trace equivalence [23] which has experimental appeal.

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References

[1] A. Arnold, A. Dicky, An algebraic characterization of transition system equivalences, Inform. and Comput. 82 (1989) 198–229.
[2] A. Arnold, A. Dicky, Equivalences and preorders of transition systems, MFCS'93, Lecture Notes in Computer Science, vol. 711, 1993, pp. 20–31.
[3] A. Arnold, I. Castellani, An algebraic characterization of observational equivalence, Theoret. Comput. Sci. 156 (1996) 289–299.
[4] J.C.M. Baeten, J.A. Bergstra, J.W. Klop, Ready-trace semantics for concrete process algebra with the priority operator, The Comput. J. 30 (6) (1987) 498–506.

[5] S.D. Brookes, C.A.R. Hoare, A.W. Roscoe, A theory of communicating sequential processes, J. ACM 31 (3) (1984) 560–599.
[6] I. Castellani, Bisimulations and abstraction homomorphisms, J. Comput. System Sci 34 (1987) 210–235.
[7] I. Castellani, P. Franceschi, U. Montanari, Labelled event structures: A model for observable concurrency, in: D. Bjorner (Ed.), Formal Description of Programming Concepts II, North-Holland, Amsterdam, 1983, pp. 383–400.
[8] E.M. Clarke, O. Grumberg, D.E. Long, Model checking and abstraction, in: Symp. on Principles of Programming Languages (POPL 92), ACM, October 1992.
[9] P. Degano, R. De Nicola, U. Montanari, A distributed operational semantics for CCS based on condition/event systems, Acta Informatica 26 (1988) 59–91.
[10] R. De Nicola, Extensional equivalences for transition systems, Acta Informatica 24 (1987) 211–237.
[11] R. De Nicola, M.C.B. Hennessy, Testing equivalences for processes, Theoret. Comput. Sci. 34 (1984) 83–133.
[12] R. De Nicola, F. Vaandrager, Three logics for branching bisimulation, Journal of ACM 42 (2) (1995) 458–487.
[13] R.J. van Glabbeek, The linear time-branching time spectrum, CONCUR'90, Lecture Notes in Computer Science, vol. 458 Springer, Berlin, 1990, pp. 278–297.
[14] R.J. van Glabbeek, The linear time-branching time spectrum II: the semantics of sequential systems with silent moves, CONCUR'93, Lecture Notes in Computer Science, vol. 715, Springer, Berlin, 1993, pp. 66–81.
[15] M. Hennessy, Acceptance trees, J. ACM 32 (4) (1985) 896–928.
[16] M. Hennessy, C. Stirling, The power of the future perfect in program logics, Inform. and Control 67 (1985) 23–52.
[17] C.A.R. Hoare, A calculus of total correctness for communicating processes, Sci. Comput. Programming 1 (1981) 49–72.
[18] C.A.R. Hoare, Communicating Sequential Processes, Prentice-Hall, Englewood Cliffs, NJ, 1985.
[19] R. Milner, Communication and Concurrency, Prentice-Hall, London, 1989.
[20] E.R. Olderog, C.A.R. Hoare, Specification-oriented semantics for communicating processes, Acta Informatica 23 (1986) 9–66.
[21] D. Park, Concurrency and automata on infinite sequences, Proc. Theoretical Computer Science, Lecture Notes in Computer Science, vol. 104, Springer, Berlin, 1981, pp. 167–183.
[22] A. Pnueli, Linear and branching structures in the semantics and logics of reactive systems, ICALP'85, Lecture Notes in Computer Science, vol. 194, Springer, Berlin, 1985, pp. 15–32.
[23] W.C. Rounds, S.D. Brookes, Possible futures, acceptances, refusals and communicating processes, in: Proc. 22nd Annual Symp. on Foundations of Computer Science, IEEE New York, Nashville, USA, 1981, pp. 140–149.